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AUTHOR(S)- J. Kranton

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ABSTRACT

This memorandum presents a detailed analysis of the stability problem associated with any attempt to passively maintain the OWS (Orbital Workshop) in a local vertical attitude during storage. The problem is far from trivial. It turns out that the aerodynamic torque plays a significant role and cannot be ignored in any meaningful discussion of the problem.

There is a paucity of papers in the open literature on the question of local vertical stabilization in the presence of aerodynamic torque. Those that have been written do not consider an asymmetrical satellite or aerodynamic torque on solar panels. In these two respects, the author believes this memorandum presents new results.

The analysis shows that a spacecraft of the OWS genre has an unstable attitude equilibrium point with respect to local vertical. Under certain conditions, not in general, the best that can be achieved is an equilibrium point which is on the borderline of instability. The orientation of the solar panels influences the degree of instability and by setting them perpendicular to the OWS roll axis the instability is minimized. Since this orientation also makes available sufficient power for the OWS during storage, it is recommended.

To accurately predict the attitude motion of a passive OWS, the effects of certain assumptions (see concluding remarks) that were made in the analysis must be quantified. This can be done only via simulation of a large number of orbits, perhaps 100, since the buildup of attitude oscillations is slow. Only if it can be shown that the attitude oscillations are either bounded or grow at an exceedingly small rate, can positive assertions be made on the acceptability of a passive OWS during storage.

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BA-146 (8-64)

BELLCOMM, INC.
1100 Seventeenth Street, N.W. Washington, D.C. 20036

SUBJECT: Passive Stability of the Local Vertical (Gravity-Gradient) Orientation of the OWS - Case 620

DATE: January 5, 1968

FROM: J. Kranton

TM-68-1022-1

TECHNICAL MEMORANDUM

INTRODUCTION

The mission concept for the OWS (Orbital Workshop) calls for it to be stored in orbit for periods of 6 months or perhaps longer. During these storage periods it is planned to keep the OWS in a local vertical, or more colloquially, gravity-gradient stabilized attitude. It will be necessary to hold this attitude and prevent tumbling in order to assure adequate power from the solar panels to support the OWS.

This memorandum presents a detailed analysis of the stability problem associated with any attempt to passively maintain the OWS in a local vertical attitude. The stability problem is far from trivial as the reader will soon appreciate. It so happens that the aerodynamic torque plays a significant role and simply cannot be ignored in any meaningful discussion of the problem.

SUMMARY OF RESULTS

In this study the assumptions have been kept to a minimum in an attempt to obtain results which need little tempering. The main assumptions are these:

1) The OWS is in a circular orbit

2) The gravity-gradient torque is produced by a pure inverse-square-law central force field

3) The air density is constant throughout the orbit.

Assumption 3) probably represents the greatest departure from reality but is necessary in order to make a complex problem manageable for analysis. The effect of this assumption on the results however seems quite small as indicated by initial data from computer simulations of the problem.

No assumption is made that the OWS is a symmetrical vehicle with equal pitch and yaw inertias.

The major results are:

1) An equilibrium attitude exists such that the aerodynamic torque is counterbalanced by the gravity-gradient torque.

2) Attitude stability of the OWS with respect to the equilibrium point depends on the orientation of the solar panels.

3) With the panels perpendicular to the OWS roll axis, the equilibrium point is, loosely speaking, on the border line separating asymptotic stability and instability.* An important finding is that any damping, no matter how small, is sufficient to produce asymptotic stability.

4) With the panels normal to the orbital plane and parallel to the OWS roll axis, the equilibrium point is unstable. However, it is possible to obtain asymptotic stability by adding slight damping on the OWS roll axis only. For example, if the maximum aerodynamic torque is 12% of the peak gravity-gradient torque, damping which produces 7.56×10^{-3} ft. lbs. per deg/sec. is sufficient.

The power available from the solar panels when oriented perpendicular to the OWS roll axis is adequate to provide the OWS power needs during storage. Moreover, the power is twice that which would be available were the panels aligned with the roll axis.

These observations coupled with the result in 3) above lead to the recommendation that the solar panels be oriented perpendicular to the roll axis during OWS storage. Furthermore, the result in 3) indicates that the inherent elasticity of the OWS structure and the solar panels may alone produce the requisite damping for asymptotic stability.

COMMENTS ON RESULTS

The results stated above are in terms of the OWS. Actually they have more general applicability.

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*This result may be sensitive to some of the assumptions and is therefore not firm. See the concluding remarks of the memorandum.

There is a paucity of papers in the open literature on the question of local vertical stabilization in the presence of aerodynamic torque.(1),(2) Those that have been written do not consider an asymmetrical satellite or aerodynamic torque on solar panels. In these two respects, the author believes this memorandum presents new results.

ACKNOWLEDGMENTS

The author was made aware of the possible instability of the local vertical attitude in the presence of aerodynamic torque by G. S. Nurre of MSFC. The motivation for the study reported here was a request by C. W. Mathews to unravel the theoretical basis for the claim that instability was possible.

The author is indebted to B. D. Elrod for many valuable exchanges of ideas and for checking the mathematical details of the analysis. He also executed the computer simulations of the complete equations of motion. Finally, thanks are owed to A. B. Filimonov who put together the program used for finding the roots of the characteristic equation (26).

PREFACE TO THE ANALYSIS

The purpose of this analysis is to demonstrate the effect of aerodynamic torque on local vertical stabilization of the OWS. It will be shown that there exists an equilibrium attitude with respect to local vertical such that the aerodynamic torque is balanced by the gravity-gradient torque. The pièce de resistance is that the stability of the equilibrium point depends significantly on the orientation of the solar panels.

A few words outlining the path we will follow in the discussion should make subsequent reading of the analysis more revealing. The first step will be to write differential equations which describe the attitude motion of the satellite with respect to local vertical coordinates. Included will be expressions for gravity-gradient and aerodynamic torques both of which are functions of attitude. In general, the equations of motion are nonlinear. In order to investigate stability we will therefore resort to perturbation analysis. This involves finding an equilibrium point, developing linear equations of motion for the perturbation variables, and then examining the roots of the characteristic equation for the linear system. To properly interpret the results of the perturbation analysis certain theorems proven by Lyapunov will be invoked.(3)

Lyapunov has shown that (a) if the solution of the linear equations is unstable, so also is the corresponding solution to the nonlinear equations, and (b) if the solution to the linear equations is asymptotically stable, so also is the corresponding solution to the nonlinear equation in the vicinity of the equilibrium point. If, however, the linear equations have a solution which is stable but not asymptotically so, no conclusion regarding the stability or even boundedness of the nonlinear equation is available.

COORDINATES

The orientation of the OWS with respect to local vertical coordinates is defined by the Euler angles, ψ, θ, and ϕ shown in Figure 1. The variable ω_o designates the orbital frequency (i.e., $\omega_0 = 2\pi/T$, where T is the orbital period).

It will be convenient for the reader's understanding of the OWS stabilization problem to associate the X axis with the OWS long (or roll) axis.

EQUATIONS OF MOTION

Euler's equations for rotational motion in spacecraft coordinates are

$$\begin{aligned}\dot{\omega}_x + K_x\,\omega_y\omega_z &= (T_x/I_x)_G + (T_x/I_x)_A \\ \dot{\omega}_y - K_y\,\omega_x\omega_z &= (T_y/I_y)_G + (T_y/I_y)_A \\ \dot{\omega}_z + K_z\,\omega_x\omega_y &= (T_z/I_z)_G + (T_z/I_z)_A\end{aligned} \tag{1}$$

where

$$K_x = (I_z - I_y)/I_x \qquad |K_x| \le 1$$

$$K_y = (I_z - I_x)/I_y \qquad |K_y| \le 1$$

$$K_z = (I_y - I_x)/I_z \qquad |K_z| \le 1$$

The subscripts x, y, z refer to the principal axes of the satellite, G refers to gravity-gradient torque, and A to aerodynamic torque. For the OWS the inertias, in slug-ft.^{2}, are approximately

$$I_x = .12006 \times 10^6,\ I_y = .93447 \times 10^6,\ I_z = .95473 \times 10^6$$

and these give

$$K_x = .16878 \quad K_y = .89320 \quad K_z = .85303$$

In terms of the Euler angles, their derivatives, and the orbital frequency, the body angular rates are

$$\omega_x = \dot{\phi} - (\dot{\psi}+\omega_0)s\theta \qquad *$$

$$\omega_y = \dot{\theta}c\phi + (\dot{\psi}+\omega_0)s\phi c\theta \qquad (2)$$

$$\omega_z = -\dot{\theta}s\phi + (\dot{\psi}+\omega_0)c\phi c\theta.$$

By differentiating (2) with respect to time, the components of $\dot{\omega}$ are obtained. These together with (2) can be substituted into (1) to give the left sides of the equations. It is not necessary for the problem at hand to carry out this step now.

GRAVITY-GRADIENT TORQUE

The gravity-gradient torque considered here is simplified by assuming that the earth possesses a pure inverse-square-law central-force gravitational field. Referred to principal axes, the gravity-gradient torque about the center of mass of the satellite is

$$T_G = 3u(z \times Iz) \qquad (3)$$

---

*In this memorandum sine and cosine will be abbreviated by s and c respectively.

where $u = GM/r^3$; G is the universal gravitation constant, and M is the mass of the earth; r is the distance between the centers of mass of the satellite and the earth; and z the unit local vertical vector expressed in body coordinates; I is the inertia matrix for the principal axes. It will be assumed that the satellite is in a circular orbit, in which case $u = \omega_0^2$.

The gravity-gradient torque is given by

$$\begin{aligned} (T_x/I_x)_G &= 3\omega_0^2 K_x (z_2 z_3) \\ (T_y/I_y)_G &= -3\omega_0^2 K_y (z_1 z_3) \qquad (4) \\ (T_z/I_z)_G &= 3\omega_0^2 K_z (z_1 z_2) \end{aligned}$$

where

$$z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} c\theta c\psi \\ -c\phi s\psi + s\phi s\theta c\psi \\ s\phi s\psi + c\phi s\theta c\psi \end{pmatrix}$$

AERODYNAMIC TORQUE

The model to be used here for the aerodynamic torque moderately approximates reality and is quite adequate for our purposes. The aerodynamic torque will be assumed to be composed of two parts, one due to the OWS and a second due to the solar panels. We will write

$$T_A = T_{A,C} + T_{A,P}$$

where, T_A, the total aerodynamic torque, is the sum of, $T_{A,C}$, the cylinder torque and, $T_{A,P}$, the panel torque. In calculating these torques, it will be assumed that (a) air density is constant, (b) the drag force acting on the OWS is directed opposite to the orbit velocity vector, (c) the OWS is cylindrically shaped, and (d) the vector between the center-of-mass and the center-of-pressure is directed along the positive X axis (i.e., roll axis) of the OWS.

Cylinder Torque

With the assumptions just mentioned, the cylinder aerodynamic torque can be written as

$$T_{A,C} = |r_{c,p}|\ |F_N|\ (1_{c,p} \times 1_{FN}) \qquad (5)$$

where

$r_{c,p}$ = vector directed from the center-of-mass to the center-of-pressure

F_N = component of drag force normal to roll axis of OWS

$1_{c,p}$ = unit vector directed along $r_{c,p}$

1_{FN} = unit vector directed along F_N

Equation (5) can be shown to reduce to

$$\begin{aligned} (T_x/I_x)_{A,C} &= 0 \\ (T_y/I_y)_{A,C} &= (T_{ml}/I_y)\ |s\gamma|\ (c) \\ (T_z/I_z)_{A,C} &= (T_{ml}/I_z)\ |s\gamma|\ (b) \end{aligned} \qquad (6)$$

where

T_{ml} = maximum value of aerodynamic torque on the cylinder

$|s\gamma| = +\ (b^2+c^2)^{1/2}$

$b = -c\phi c\psi - s\phi s\theta s\psi$

$c = -s\phi c\psi + c\phi s\theta s\psi$

A key step is now to express T_{m1} as a constant, α, times the peak gravity-gradient torque. That is, we let

$$T_{m1} = \alpha |T_{G,m}| \tag{7}$$

where

$|T_{G,m}|$ = maximum gravity-gradient torque, which will be defined below.

<u>Solar Panel Torque</u>

The solar panel torque will be a function of the panel orientation with respect to the OWS. In Figure 2* a panel angle p is defined. With p = 0, the plane of the panels contains the OWS roll axis, and with p = -90^o, the plane of the panels is perpendicular to the OWS roll axis.

The solar panel torque can be written as

$$T_{A,P} = |r_{p,p}| \; |F_P| \; (1_{N,P} \cdot 1_F)^2 \; (1_{p,p} \times 1_{N,P})^{**} \tag{8}$$

where

$r_{p,p}$ = vector directed from the OWS center-of-mass to the panel center-of-pressure

F_P = total drag force on panels

$1_{p,p}$ = unit vector directed along $r_{p,p}$

$1_{N,P}$ = unit vector directed along component of F_P normal to panels

1_F = unit vector directed along F_P

---

*The principal axes of the OWS are actually not aligned as depicted in Figure 2. This point is not critical to the results developed in this study.

**This equation assumes that the aerodynamic force on the panels is normal to the plane of the panels.

Equation (8) can be shown to reduce to

$$
\begin{aligned}
(T_x/I_x)_{A,P} &= 0 \\
(T_y/I_y)_{A,P} &= 0 \qquad\qquad (9) \\
(T_z/I_z)_{A,P} &= (T_{m2}/I_z)\ (-m|m|cp)
\end{aligned}
$$

where

T_{m2} = maximum value of aerodynamic torque on the solar panels,

$m = -sp\ (c\theta s\psi) + cp\ (c\phi c\psi + s\phi s\theta s\psi)$.

T_{m2} will be expressed as a constant, β, times the maximum gravity-gradient torque. That is,

$$T_{m2} = \beta |T_{G,m}| \qquad (10)$$

the sum of (6) and (9) completes the model of the aerodynamic torque.

PERTURBATION ANALYSIS

With a little heuristic insight, we anticipate an equilibrium attitude at the orientation $\phi = 0$, $\theta = 0$, $\psi = \psi_s$ such that the aerodynamic torque is balanced by the gravity-gradient torque. Below we will determine ψ_s explicitly.

If only small perturbations about the equilibrium orientation are considered, then

$$\phi = \delta\phi,\ \theta = \delta\theta,\ \psi = \psi_s + \delta\psi$$

Linearizing equations (2), (4), (6) and (9) about the equilibrium point we obtain

$$
\begin{aligned}
\omega_x &= \delta\dot{\phi}-\omega_0\delta\theta, & \dot{\omega}_x &= \delta\ddot{\phi}-\omega_0\delta\dot{\theta} \\
\omega_y &= \delta\dot{\theta}+\omega_0\delta\phi, & \dot{\omega}_y &= \delta\ddot{\theta}+\omega_0\delta\dot{\phi} \\
\omega_z &= \delta\dot{\psi}+\omega_0 \quad, & \dot{\omega}_z &= \delta\ddot{\psi}
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
\omega_y\omega_z &= \omega_0\delta\dot{\theta}+\dot{\omega}_0{}^2\delta\phi \\
\omega_x\omega_z &= \omega_0\delta\dot{\phi}-\omega_0{}^2\delta\theta \\
\omega_x\omega_y &= 0
\end{aligned}
$$

$$
\left.
\begin{aligned}
(T_x/I_x)_G &= -3\omega_0{}^2K_x(s^2\psi_s\delta\phi+s\psi_s c\psi_s\delta\Theta) \\
(T_y/I_y)_G &= -3\omega_0{}^2K_y(c\psi_s s\psi_s\delta\phi+c^2\psi_s\delta\Theta) \\
(T_z/I_z)_G &= -3\omega_0{}^2K_z((c^2\psi_s-s^2\psi_s)\delta\psi+c\psi_s s\psi_s)
\end{aligned}
\right\}
\tag{12}
$$

$$
\left.
\begin{aligned}
(T_x/I_x)_{A,C} &= 0 \\
(T_y/I_y)_{A,C} &= \left(\frac{\alpha|T_{G,m}|}{I_y}\right)(-c^2\psi_s\delta\phi+c\psi_s s\psi_s\delta\Theta) \\
(T_z/I_z)_{A,C} &= \left(\frac{\alpha|T_{G,m}|}{I_z}\right)(-c^2\psi_s+2s\psi_s c\psi_s\delta\psi)
\end{aligned}
\right\}
\tag{13}
$$

$$\left.\begin{aligned}(T_x/I_x)_{A,P} &= 0\\(T_y/I_y)_{A,P} &= 0\\(T_z/I_z)_{A,P} &= \left(\frac{\beta|T_{G,m}|}{I_z}\right)(-c^2(\psi_s+p)+2s(\psi_s+p)c(\psi_s+p)\delta\psi)cp\end{aligned}\right\}\quad(14)$$

Substituting (11), (12), (13) and (14) into (1) we obtain

$$\delta\ddot{\phi} + \delta\phi\omega_0{}^2K_x(1+3s^2\psi_s) + \delta\dot{\theta}[-\omega_0(1-K_x)] + \delta\theta(3\omega_0{}^2K_x s\psi_s c\psi_s) = 0 \qquad (15a)$$

$$\delta\ddot{\theta} + \delta\theta[\omega_0{}^2K_y(1+3c^2\psi_s) - \left(\frac{\alpha|T_{G,m}|}{I_y}\right) c\psi_s s\psi_s] + \delta\dot{\phi}\omega_0(1-K_y) + \delta\phi(3\omega_0{}^2K_y c\psi_s s\psi_s + \left(\frac{\alpha|T_{G,m}|}{I_y}\right) c^2\psi_s) = 0 \qquad (15b)$$

$$\delta\ddot{\psi} + \delta\psi[3\omega_0{}^2K_z(c^2\psi_s - s^2\psi_s) - \left(\frac{\alpha|T_{G,m}|}{I_z}\right) 2s\psi_s c\psi_s - \left(\frac{\beta|T_{G,m}|}{I_z}\right) 2s(\psi_s+p)c(\psi_s+p)cp]$$

$$= -3\omega_0{}^2K_z c\psi_s s\psi_s - \left(\frac{\alpha|T_{G,m}|}{I_z}\right) c^2\psi_s - \left(\frac{\beta|T_{G,m}|}{I_z}\right) c^2(\psi_s+p)cp \qquad (15c)$$

SOLUTION FOR EQUILIBRIUM POINT

The left sides of (15) are constant-coefficient linear differential equations for the perturbation variables $\delta\phi$, $\delta\theta$, and $\delta\psi$. The right sides of (15) can be viewed as the driving functions for the linear system.

By definition, the condition for equilibrium is that all the perturbation variables and their derivatives be zero. Consequently, we may equate the right side of (15c) to zero and thereby obtain an explicit expression for ψ_s. That is,

$$-3\omega_0{}^2 K_z c\psi_s s\psi_s - \left(\frac{\alpha|T_{G,m}|}{I_z}\right) c^2\psi_s - \left(\frac{\beta|T_{G,m}|}{I_z}\right) c^2(\psi_s+p)cp = 0 \quad (16)$$

For a given panel angle, p, (16) can be solved for ψ_s provided we know $|T_{G,m}|$. The expression for $|T_{G,m}|$ is obtained by noting that for stable behavior with respect to the local vertical in the absence of aerodynamic torque it is required that[4]

$$I_z \gtrless I_y > I_x \quad (17)$$

As a result,

$$|T_{G,m}| = \frac{3}{2}\,\omega_0{}^2 I_y K_y \quad (18)$$

In what follows we will investigate the stability problem for p equal to 0° and -90°. These values of p are chosen because the analysis can be expedited with the additional benefit of providing insight into the nature of the stability problem. At p = 0° the solution to (16) is

$$\tan\psi_s = -\frac{(\alpha+\beta)}{2}\,\frac{I_z-I_x}{I_y-I_x} = -\frac{(\alpha+\beta)}{2}\,\frac{I_y K_y}{I_z K_z} \quad (19)$$

and at $p = -90^\circ$

$$\tan \psi_s = - \frac{\alpha}{z} \frac{I_z - I_x}{I_y - I_x} = - \frac{\alpha}{z} \frac{I_y K_y}{I_z K_z} \tag{20}$$

In view of (17) we see that ψ_s is negative, a fact which our earlier heuristic insight told us should be the case.

STABILITY ANALYSIS

As a first step in investigating the stability of the attitude motion with respect to the equilibrium point, we find the Laplace transform of (15) and write the result in matrix form

$$\begin{bmatrix} s^2+K_x\omega_0^2(1+3s^2\psi_s) & -s(1-K_x)\omega_0+3\omega_0^2K_x s\psi_s c\psi_s & 0 \\ \begin{array}{l} s(1-K_y)\omega_0+3\omega_0^2K_y c\psi_s s\psi_s \\ +\alpha\frac{3}{2}\omega_0^2K_y c^2\psi_s \end{array} & \begin{array}{l} s^2+K_y\omega_0^2(1+3c^2\psi_s) \\ -\alpha\frac{3}{2}\omega_0^2K_y c\psi_s s\psi_s \end{array} & 0 \\ 0 & 0 & \begin{array}{l} s^2+3\omega_0^2K_z(c^2\psi_s-s^2\psi_s) \\ -\alpha 3\omega_0^2\,\frac{I_yK_y}{I_z}\,s\psi_s c\psi_s \\ -\beta 3\omega_0^2\,\frac{I_yK_y}{I_z}\,s(\psi_s+p)c(\psi_s+p)cp \end{array} \end{bmatrix} \begin{bmatrix} \delta\phi(s) \\ \delta\Theta(s) \\ \delta\psi(s) \end{bmatrix} = 0 \quad (21)$$

The characteristic equation for the $\delta\psi$ motion is

$$s^2 + 3\omega_0{}^2[K_z(c^2\psi_s - s^2\psi_s) - \frac{I_y K_y}{I_z}(\alpha s\psi_s c\psi_s + \beta s(\psi_s + p)c(\psi_s + p)cp)] = 0 \quad (22)$$

and the characteristic equation corresponding to the $\delta\phi$, $\delta\Theta$ motion is

$$s^4 + s^2\omega_0{}^2[1 + 3K_x s^2\psi_s + 3K_y c^2\psi_s + K_x K_y - \alpha\tfrac{3}{2}K_y c\psi_s s\psi_s]$$

$$+ s\omega_0{}^3[3(K_y - K_x)c\psi_s s\psi_s + \alpha\tfrac{3}{2}K_y(1 - K_x)c^2\psi_s]$$

$$+ \omega_0{}^4[4K_x K_y(1 - \alpha\tfrac{3}{2}c\psi_s s\psi_s)] = 0 \quad (23)$$

Equation (22) is of the form $s^2 + a^2 = 0$ so that its roots are pure imaginary. Consequently, any damping, however small, on the OWS Z axis will make the $\delta\psi$ motion asymptotically stable with respect to the equilibrium point ψ_s.

As to equation (23), we note that there is no s^3 term. This means that the sum of the roots is zero and we must concern ourselves with possible instability (i.e., roots with positive real parts). In fact, if the coefficient of the s term is not zero we are assured of roots with positive real parts. We will now investigate the nature of the roots for panel angles of 0° and -90°.

Panel Angle = -90°

With p = -90°, ψ_s is given by (20). This equation can be solved for $\alpha/2$ and the result substituted into the expressions for the coefficients of (23). When this is done it can be shown, after some algebraic manipulations, that the coefficient of the s term is zero for all K_x, K_y. As a result (23) becomes

$$s^4 + s^2\omega_0{}^2[1+3K_x s^2\psi_s+3K_y c^2\psi_s+K_x K_y-\alpha\tfrac{3}{2}K_y c\psi_s s\psi_s]$$

$$+ \omega_0{}^4[4K_x K_y(1-\alpha\tfrac{3}{2}c\psi_s s\psi_s)] = 0 \qquad (24)$$

This equation is of the form

$$s^4 + 2bs^2 + c = 0$$

Any of the following three conditions is sufficient for the roots to have positive real parts

$$b < 0,\ c < 0,\ b^2-c < 0$$

If none of these inequalities hold, and there are no zero roots, then all the roots are pure imaginary. Indeed this is the case for the OWS. The table below gives the values of the roots and ψ_s for representative values of α.

α	Roots	ψ_s
.01	$\pm j\ .4056\omega_0,\ \pm j\ 1.915\omega_0$	$-.294^\circ$
.02	$\pm j\ .4056\omega_0,\ \pm j\ 1.915\omega_0$	$-.587^\circ$
.03	$\pm j\ .4057\omega_0,\ \pm j\ 1.915\omega_0$	$-.881^\circ$

When the roots of the characteristic equation for the linear system are pure imaginary, Lyapunov theory offers no conclusion regarding the stability or even boundedness of the nonlinear equation. This is a theoretical result. Computer simulations of the nonlinear equations however show that the attitude motions are bounded (i.e., oscillatory with respect to the equilibrium point). Moreover, it will be shown that any small amount of damping will make the $\delta\phi$ and $\delta\theta$ motion asymptotically stable with respect to the equilibrium attitude.

Panel Angle = 0°

We will show for this case that the coefficient of the s term in (23) is negative. This is sufficient to demonstrate that the roots have positive real parts.

With p = 0°, ψ_s is given by (19). This equation can be solved for $(\alpha+\beta)/2$. Now, add and substract to the coefficient of the s term in (23) $\beta\frac{3}{2}K_y(1-K_x)c^2\psi_s$. The coefficient becomes

$$\omega_0^2[3(K_y-K_x)c\psi_s s\psi_s + \frac{(\alpha+\beta)}{2} 3K_y(1-K_x)c^2\psi_s] - \omega_0^2[\beta\frac{3}{2}K_y(1-K_x)c^2\psi_s]$$

By substituting the solution for $(\alpha+\beta)/2$ obtained from (19), it can be shown that the left hand term is zero for all K_x, K_y. As a result (23) becomes

$$s^4 + s^2\omega_0^2[1+3K_x s^2\psi_s+3K_y c^2\psi_s+K_xK_y-\alpha\frac{3}{2}K_y c\psi_s s\psi_s]$$

$$- s\omega_0^3[\beta\frac{3}{2}K_y(1-K_x)c^2\psi_s]$$

$$+ \omega_0^4[4K_xK_y(1-\alpha\frac{3}{2}c\psi_s s\psi_s)] = 0 \qquad (25)$$

and the coefficient of the s term is negative for all K_x, K_y.

The table below gives the values of the roots and ψ_s for α = .02 and representative values of β.

β	Roots		ψ_s
.10	$-(1/1.003T)$, $\pm j\ 1.913\omega_0$	$+(1/1.003T)$, $\pm j\ .4060\omega_0$	-3.52°
.15	$-(1/.671T)$, $\pm j\ 1.911\omega_0$	$+(1/.671T)$, $\pm j\ .4061\omega_0$	-4.98°
.20	$-(1/.503T)$, $\pm j\ 1.909\omega_0$	$+(1/.503T)$, $\pm j\ .4062\omega_0$	-6.43°
.25	$-(1/.403T)$, $\pm j\ 1.906\omega_0$	$+(1/.403T)$, $\pm j\ .4064\omega_0$	-7.88°
.30	$-(1/.338T)$, $\pm j\ 1.902\omega_0$	$+(1/.338T)$, $\pm j\ .4064\omega_0$	-9.31°

$T = \frac{2\pi}{\omega_0}$ = orbital period.

It will be shown below that by adding damping to the OWS roll axis only, it is possible to move the roots to the left-half plane and thereby obtain asymptotic stability with respect to the equilibrium point.

EFFECTS OF DAMPING

In this section we will consider the effects of damping from a mathematical viewpoint without regard to the details of producing the damping. It is assumed that the damping to be discussed can be achieved in practice, at least approximately.

First, observe that in (21) if we make the substitution $s = \omega_0\lambda$, ω_0^2 can be factored out and the resulting characteristic equation in λ is the same as (23) except that the ω_0^2, ω_0^3, and ω_0^4 multipliers of the coefficients no longer appear.

We focus on (21) again, in particular on the (1,1) and (2,2) elements of the matrix. In the (1,1) element define

$$\omega_1^2 = K_x(1+3s^2\psi_s)$$

and in the (2,2) element define

$$\omega_2^2 = K_y(1+3c^2\psi_s) - \alpha\frac{3}{2}K_y c\psi_s s\psi_s$$

We will introduce damping by adding a term $2\zeta_1\omega_1$ to the (1,1) element and $2\zeta_2\omega_2$ to the (2,2) element, and then attempt to find values for ζ_1 and ζ_2 which will produce roots in the left-half plane. When the damping terms are introduced the characteristic equation for the $\delta\phi$, $\delta\Theta$ motion becomes

$$\lambda^4 + \lambda^3(2\zeta_1\omega_1+2\zeta_2\omega_2)$$
$$+ \lambda^2[1+3K_x s^2\psi_s+3K_y c^2\psi_s+K_xK_y - \alpha\tfrac{3}{2}K_y c\psi_s s\psi_s+(2\zeta_1\omega_1)(2\zeta_2\omega_2)]$$
$$+ \lambda[3(K_y-K_x)c\psi_s s\psi_s+\alpha\tfrac{3}{2}K_y(1-K_x)c^2\psi_s+2\zeta_1\omega_1\omega_2{}^2+2\zeta_2\omega_2\omega_1{}^2]$$
$$+ [4K_xK_y(1-\alpha\tfrac{3}{2}c\psi_s s\psi_s)] = 0 \tag{26}$$

To investigate the requirements on ζ_1 and ζ_2 to produce roots with negative real parts, the Hurwitz criterion is applied to the coefficients of (26). The details of this analysis are lengthy and we will not burden the reader with them. The results for the OWS are as follows.

Panel Angle = -90°

Any positive ζ_1 or ζ_2, however small, is sufficient to give roots with negative real parts and thereby produce asymptotic stability.

Panel Angle = 0°

In this case the result is that a positive ζ_1 and zero ζ_2 are sufficient, and the minimum value of ζ_1 is given by the requirement that the coefficient of the λ term be positive. For the OWS the minimum value of ζ_1 is .039 with α = .02 and β = .10. It is also possible to produce stability with positive ζ_2 and zero ζ_1. However, in this case the value of ζ_2 required is extremely high.

The interpretation of these results is that damping of the OWS about its roll axis is sufficient to produce asymptotic stability. The value of ζ_1 = .039 corresponds to damping that produces a torque of 7.56×10^{-3} ft.-lbs. per deg/sec.

COMMENTS ON ANOTHER OWS SOLAR PANEL ORIENTATION

We have considered orientations in which the Z axis and the plane of the panels are perpendicular to the orbit plane. For the sake of completeness we must comment on another orientation in which the Y axis (see Figure 2) is perpendicular to the orbit plane and the panel angle is adjusted to optimize the solar energy incident on the panels.(7) This method attains, under certain conditions, the maximum solar energy in a local vertical attitude. It is not difficult to see that the aerodynamic drag makes this attitude unstable. The mathematical analysis of this instability is far more complicated than that given above. It is not deemed worthwhile to undertake the analysis at this time since the OWS storage power requirements, as presently estimated, can be met with the Z axis perpendicular to the orbit plane and the panels turned to $p = -90^{\circ}$.

CONCLUDING REMARKS

Although a complicated mathematical route has been traversed to obtain our results, there are still outstanding questions. These questions concern the effects the assumptions have on the outcome of the analysis. Until these effects are quantified the long term motion of a passive OWS in storage is open to doubt. Specifically, it is necessary to evaluate the effects of non-uniform air density (diurnal bulge), non-circular orbit, earth's oblateness, and the offsets of the center-of-pressure and center-of-mass from the OWS roll axis.

Other factors which must be taken into account are the imprecise knowledge of the location of the principal axes and of the air density and its variations. The influence of these factors is that the set of initial conditions used to investigate the motion of the OWS must be sufficiently large to encompass all the uncertainty.

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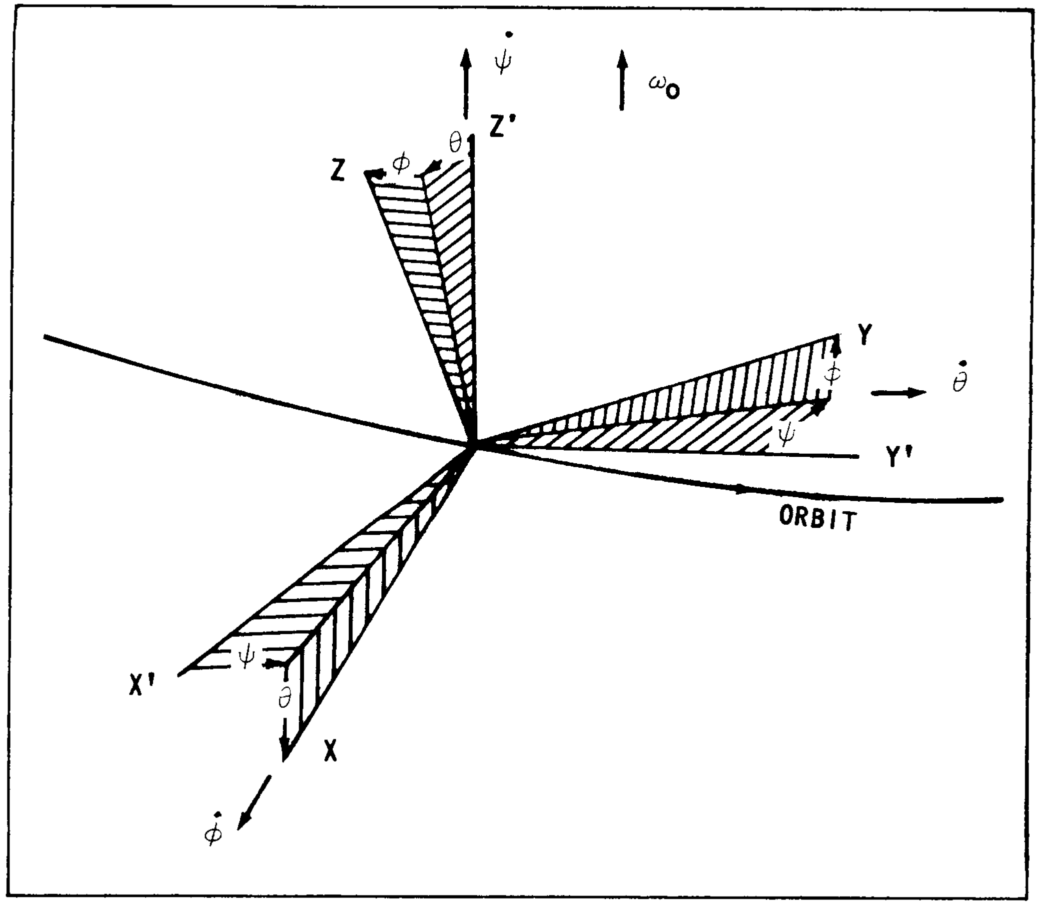
Attachments
Figures 1 and 2
References



X'Y'Z' = LOCAL VERTICAL COORDINATES
X' = UPWARD LOCAL VERTICAL DIRECTION
Z' = NORMAL TO ORBIT PLANE

XYZ = SPACECRAFT PRINCIPAL AXES

XYZ IS OBTAINED FROM X'Y'Z' BY ROTATING THROUGH THE EULER SEQUENCE ψ, θ, ϕ

FIGURE 1

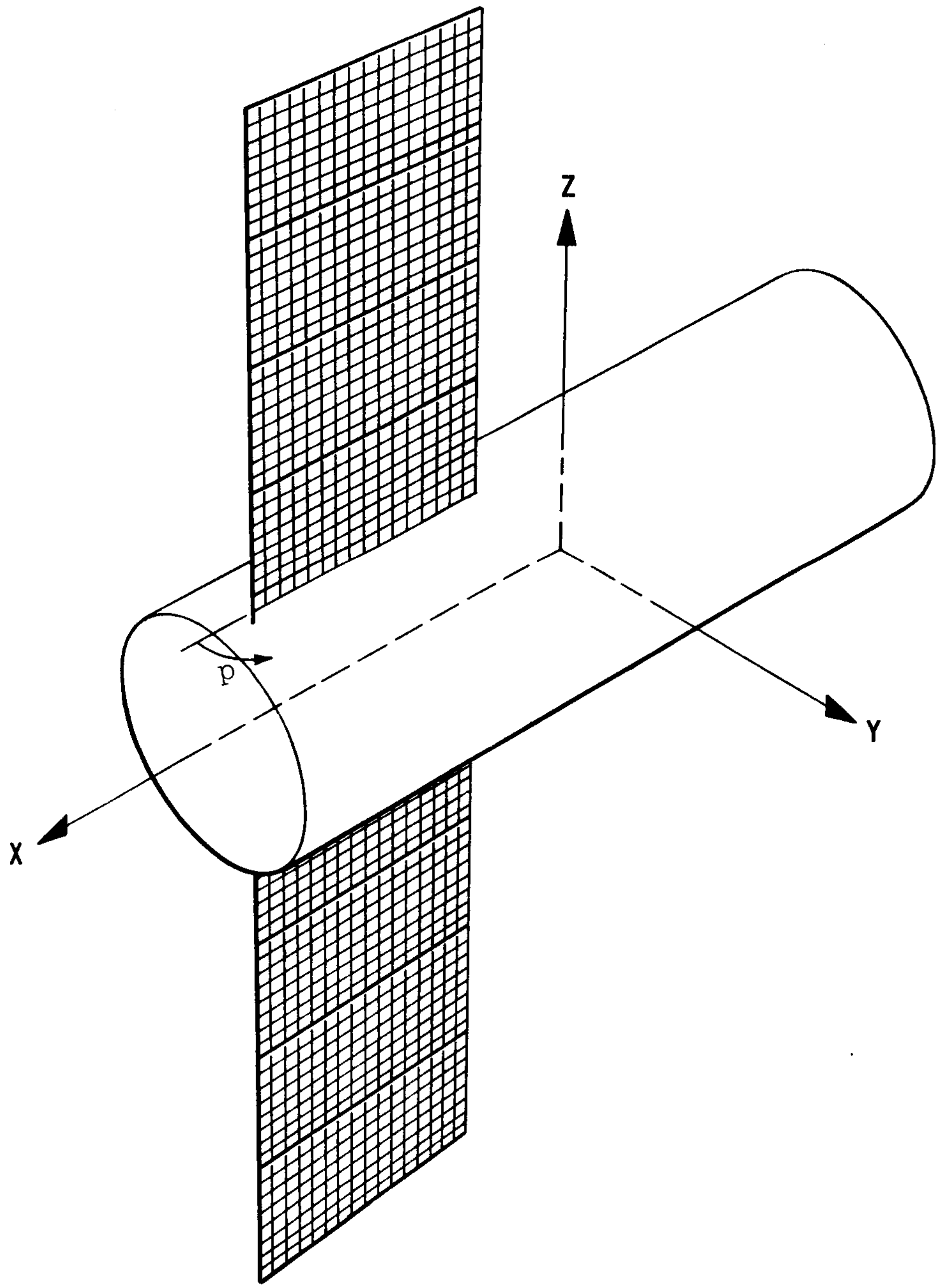


FIGURE 2

REFERENCES

1. Garber, T. B., "Influence of Constant Disturbing Torques on the Motion of Gravity Gradient Stabilized Satellites", AIAA J., 968-969, April 1963.

2. Meirovitch, L. and Wallace, F. B., "On the Effect of Aerodynamic and Gravitational Torques on the Attitude Stability of Satellites", AIAA J., 2196-2202, December 1966.

3. Cesari, L., "Asymptotic Behavior and Stability Problems in Ordinary Differential Equations", Academic Press, New York, 1963.

4. DeBra, D. B. and Delp, R. H., "Satellite Stability and Natural Frequencies in a Circular Orbit", J. Astr. Sci. 8, 14-17, 1961.

5. Hough, W. W., "The Effect of Launch Time on the Performance of a Solar Array/Battery Electrical Power System", Bellcomm Technical Memorandum, TM-67-1022-3, July 11, 1967.